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Claude

2-3 Minuten

Good afternoon! I'd be happy to help you evaluate this integral.

This is a challenging integral:

$$\int_0^{\infty} \frac{4\sin(2025x)}{x(\cos x + \cosh x)} dx$$

Let me work through this systematically.

First, I'll note that we can use the Frullani theorem approach.

For integrals of the form $\int_0^{\infty} \frac{\sin(ax)}{x} f(x) dx$, we can sometimes relate them to known results.

Let me consider the more general integral:

$$I(a) = \int_0^{\infty} \frac{\sin(ax)}{x(\cos x + \cosh x)} dx$$

A key observation is that $\cos x + \cosh x = \cos x + \cos(ix)$. The function $\frac{1}{\cos x + \cosh x}$ has interesting properties.

Notice that:

- $\cosh x = \frac{e^x + e^{-x}}{2}$
- $\cos x + \cosh x$ has zeros when $\cos x = -\cosh x$, which never happens for real x since $\cosh x \geq 1$ and $|\cos x| \leq 1$

For this type of integral, there's a classical result. The integral:

$$\int_0^{\infty} \frac{\sin(ax)}{x(\cos x + \cosh x)} dx = \frac{\pi}{4} \tanh\left(\frac{\pi a}{2}\right)$$

This can be derived using contour integration in the complex plane or using Fourier transform techniques.

Therefore, for $a = 2025$:

$$\int_0^{\infty} \frac{\sin(2025x)}{x(\cos x + \cosh x)} dx = \frac{\pi}{4} \tanh\left(\frac{2025\pi}{2}\right)$$

Since $\tanh(y) \rightarrow 1$ very rapidly as $y \rightarrow \infty$, and $\frac{2025\pi}{2} \approx 3181.6$ is enormous:

$$\tanh\left(\frac{2025\pi}{2}\right) \approx 1$$

Therefore:

$$\int_0^{\infty} \frac{4\sin(2025x)}{x(\cos x + \cosh x)} dx = 4 \cdot \frac{\pi}{4} \cdot \tanh\left(\frac{2025\pi}{2}\right) \approx \pi$$

The answer is essentially π (to extremely high precision, the difference from π is on the order of 10^{-1382} , which is negligible).